Matrix Factorizations, Old and New

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An old factorization is A = LU, expressing elimination in a neat way. When there are row exchanges, a permutation matrix is needed. SIAM would put it before L, algebraists would prefer A = LPU (where P is unique). The new question is for doubly infinite matrices because how can elimination start ? If the infinite matrix is banded we want to show that still

A = LPU = (lower triangular) (permutation) (upper triangular).

A special class contains the matrices that are banded and also have a banded inverse – NOT typical of finite differences and finite elements, but interesting and useful. We look for a factorization when $A_{ij} = 0$ and also $inv(A)_{ij} = 0$ for $|i - j| \leq w$. Block diagonal matrices have this property ! We factor A into BC using block diagonal B and C.

And some thoughts about the role of linear algebra.......